GRAPH $r = 1 + \cos 2\theta$ USING SYMMETRY

POLAR AXIS $(r, -\theta)$: $r = 1 + \cos 2(-\theta)$

$r = 1 + \cos(-2\theta)$

$r = 1 + \cos 2\theta \leftarrow$ SYM

POLAR AXIS ✓ $(r, -\theta)$ ~~$(-r, \pi - \theta)$~~

POLE ~~$(-r, \theta)$~~ ✓ $(r, \pi + \theta)$

$\theta = \frac{\pi}{2}$? $(-r, -\theta)$ $(r, \pi - \theta)$

$\cos(-x) = \cos x$

POLE $(r, \pi + \theta)$: $r = 1 + \cos 2(\pi + \theta)$

$r = 1 + \cos(2\pi + 2\theta)$

$r = 1 + \cos 2\pi \cos 2\theta - \sin 2\pi \sin 2\theta$

$r = 1 + \cos 2\theta \leftarrow$ SYM

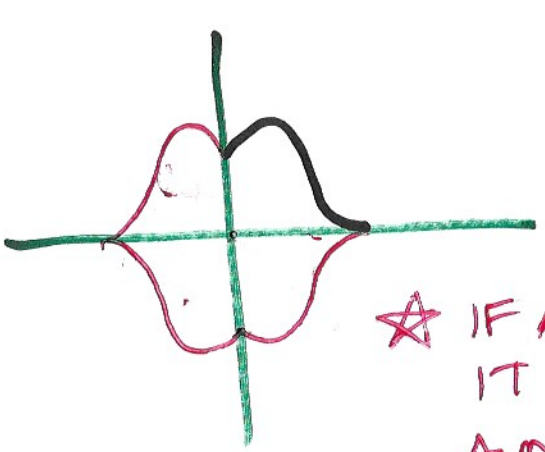
$\cos(A+B) = \cos A \cos B - \sin A \sin$

YOU COULD RUN SYM TEST TO DETERMINE IF GRAPH IS SYM OVER $\theta = \frac{\pi}{2}$

BUT, IF A GRAPH IS SYM OVER POLAR AXIS + POLE

IT IS AUTOMATICALLY SYM OVER $\theta =$

SO, YOU DON'T NEED TO RUN THE SYM TESTS FOR $\theta = \frac{\pi}{2}$



☆ IF A GRAPH IS SYM IN 2 WAYS,

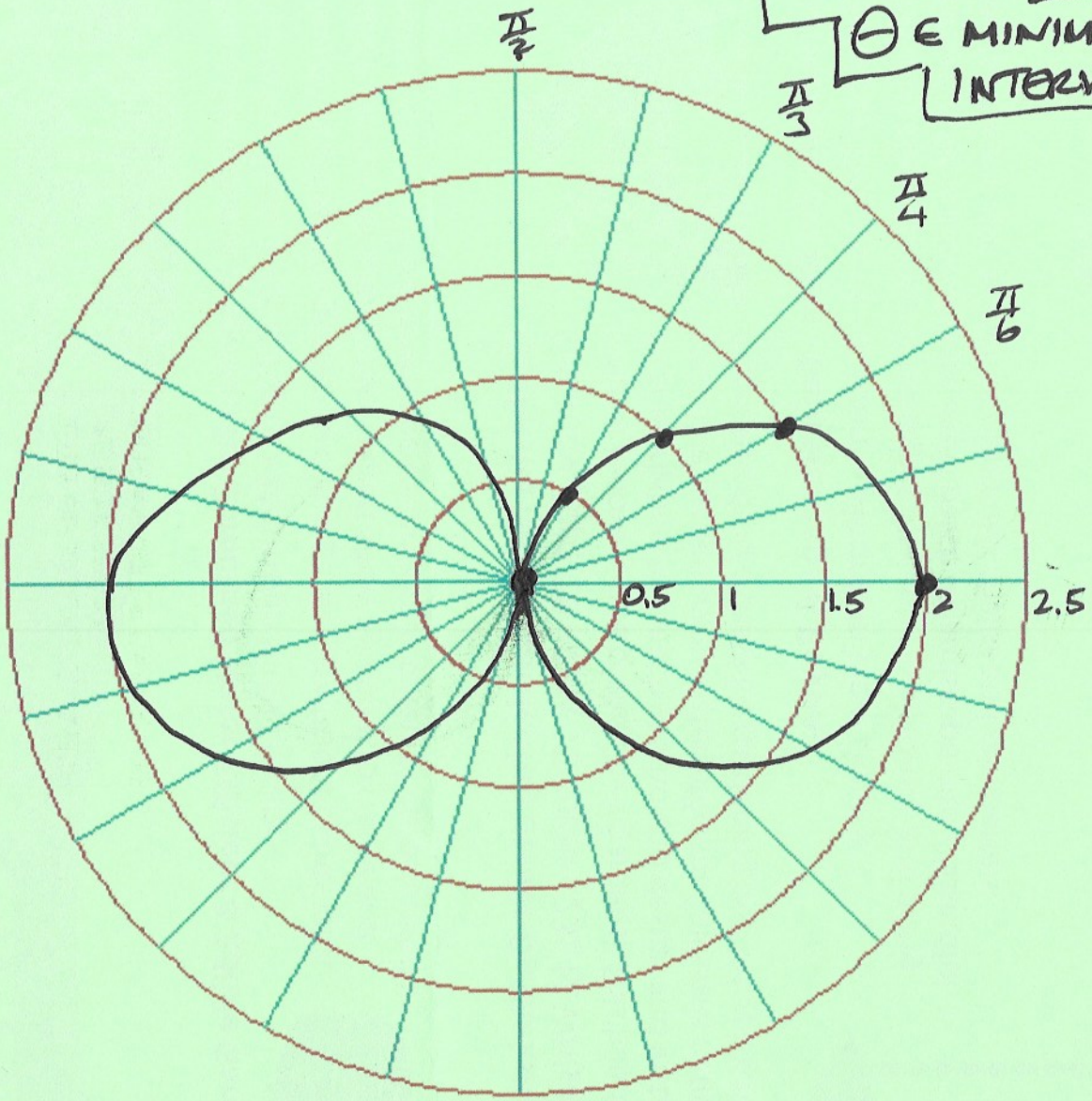
IT IS AUTOMATICALLY SYMMETRIC IN ALL 3 W

AND THE MINIMUM INTERVAL IS $\theta \in [0, \frac{\pi}{2}]$

$r = 1 + \cos 2\theta$

θ	$r = 1 + \cos 2\theta$	(r, θ)
0	$1 + \cos 2(0) = 1 + \cos 0 = 1 + 1 = 2$	$(2, 0)$
$\frac{\pi}{6}$	$1 + \cos 2(\frac{\pi}{6}) = 1 + \cos \frac{\pi}{3} + 1 + \frac{1}{2} = 1\frac{1}{2}$	$(1.5, \frac{\pi}{6})$
$\frac{\pi}{4}$	$1 + \cos 2(\frac{\pi}{4}) = 1 + \cos \frac{\pi}{2} = 1 + 0 = 1$	$(1, \frac{\pi}{4})$
$\frac{\pi}{3}$	$1 + \cos 2(\frac{\pi}{3}) = 1 + \cos \frac{2\pi}{3} = 1 - \frac{1}{2} = \frac{1}{2}$	$(0.5, \frac{\pi}{3})$
$\frac{\pi}{2}$	$1 + \cos 2(\frac{\pi}{2}) = 1 + \cos \pi = 1 - 1 = 0$	$(0, \frac{\pi}{2})$

$r = 1 + \cos 2\theta$



FOR WHAT θ ∈ MINIMUM INTERVAL → WHEN DOES $r = 1 + \cos 2\theta$ PASS THROUGH POLE?

$\theta \in [0, \frac{\pi}{2}]$ ↑ $r = 0$

$0 = 1 + \cos 2\theta$

$\cos 2\theta = -1$

LET $t = 2\theta$

$\cos t = -1$

$0 \le \theta \le \frac{\pi}{2}$

$0 \le 2\theta \le \pi$

$0 \le t \le \pi$

$t = \pi$

$2\theta = \pi$

$\theta = \frac{\pi}{2}$

FOR WHAT VALUES OF θ $\in$ MINIMUM INTERVAL DOES $r = 1 + 2\sin 2\theta$ GO THROUGH THE POLE ?

(IT TURNS THAT THIS GRAPH IS SYM OVER POLE)

↳ RUN SYM TESTS YOURSELF TO CONFIRM

SO, MINIMUM INTERVAL $\theta \in [0, \pi]$

OR $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

WE CHOOSE $\theta \in [0, \pi]$

$$0 = 1 + 2\sin 2\theta$$

$$\sin 2\theta = -\frac{1}{2}$$

$0 \le \theta \le \pi$

$0 \le 2\theta \le 2\pi$

$$\tfrac{1}{2}(2\theta) = \tfrac{1}{2}\left(\frac{7\pi}{6}\right), \left(\frac{11\pi}{6}\right)\tfrac{1}{2}$$

$$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}$$ ← PLOT A POINT AT POLE CORRESPONDING TO $\theta = \frac{7\pi}{12}, \frac{11\pi}{12}$

BETWEEN $\frac{\pi}{2}, \frac{2\pi}{3}$

BETWEEN $\frac{5\pi}{6}, \pi$

CONNECT POINT FOR $\theta = \frac{\pi}{2}$ TO POLE @ $\theta = \frac{7\pi}{12}$

TO POINT FOR $\theta = \frac{2\pi}{3}$

$r = a + b\sin\theta$ OR $a + b\cos\theta$ $a, b \in \mathbb{R}$

↑ ONLY θ

1. IMPACT OF $|a|, |b|$ ON SHAPE
2. IMPACT OF $\sin\theta$ VS $\cos\theta$
3. IMPACT OF SIGNS OF a, b

→ 10.8 POLAR GRAPHING & POLAR GRAPHING

SEE WEBSITE HANDOUT +

PLAY WITH DESMOS.COM